

Analysis of a Discrete Parametric Markov-Chain Model of a Two Unit Cold Standby System With Repair Machine failure

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Abstract— This paper deals with the cost-benefit analysis of a two-identical unit cold standby system model assuming two modes- normal and total failure of each unit. To repair a failed unit a repair machine (RM) is required which is good initially and can't fail until it begins functioning. During the repair of a failed unit, the RM may also fail. A single repairman is always available with the system to repair a failed unit as well as the failed RM. The random variables denoting the failure and repair times of RM and the units are independent of discrete nature having geometric distributions with different parameters. The various measures of system effectiveness are obtained by using regenerative point technique.

Keywords: Transition probability, mean sojourn time, regenerative point, reliability, MTSF, availability of system, busy period of repairman.

1 Introduction

TWO unit redundant systems have been widely studied in the literature of reliability due to their frequent and significant use in modern business and industries. Various authors [1, 2, 3, 4, 7, 9] have studied two-unit parallel and standby redundant system models using the concepts of common cause failure, inspection for repair/post-repair, two types of repairman, waiting time of repairman and preparation for repair. The common assumption considered in analyzing these models is that the machine/ device used for repairing a failed unit remains perfect forever. In real existing situations, this assumption is not always practicable as the RM may also have a specified reliability and can fail during the repair process of a failed unit. For example: In the case of nuclear reactors, marine equipments etc, the robots are used for the repair of such type of systems. It is evident that a robot again being a machine may fails while performing its intended task. In this case obviously the repairman first repairs the failed RM and then takes up the failed unit for repair. Keeping this fact in view, authors [5, 6, 8] analyzed the system models assuming that the RM may also fail. All the above authors have obtained various measures of system effectiveness under continuous parametric Markov-Chain.

The purpose of the present paper is to analyze a two-identical unit cold standby system model with RM failure under **discrete parametric Markov-Chain i.e. failure and repair times of RM and units are taken as discrete random variables having geometric distributions with different parameters**. The following economic related measures of system effectiveness are obtained by using regenerative point technique-

i) Transition probabilities and mean sojourn times in vari-

- ous states.
- ii) Reliability and mean time to system failure.
 - iii) Point-wise and steady-state availabilities of the system as well as expected up time of the system during interval $(0, t)$.
 - iv) Expected busy period of the repairman during time interval $(0, t)$.
 - v) Net expected profit earned by the system during a finite interval and in steady-state.

2 Model Description and Assumption

- i) The system comprises of two-identical units. Initially, one unit is operative and other is kept into cold standby.
- ii) Each unit of the system has two modes- Normal (N) and Total failure (F).
- iii) To repair a failed unit a RM is required. Initially, the RM is good and it can't fail unless it begins operative.
- iv) A single repairman is always available with the system to repair a failed unit as well as failed RM.
- v) The RM may fail, during the repair of a failed unit. In this case, the repair of the failed unit is discontinued to start the repair of RM.
- vi) The repaired units and RM work as good as new.

The phenomena of discrete failure and repair time distributions may be observed in the following situation.

Let the continuous time period $(0, \infty)$ is divided as $0, 1, 2, \dots, n, \dots$ of equal distance on real line and the probability of failure of a unit during time $(i, i+1)$; $i = 0, 1, 2, \dots$ is p , then the probability that the unit will fail during $(t, t+1)$ i.e. after passing successfully t intervals of time, is given by $p(1-p)^t$; $t = 0, 1, 2, \dots$

This is the p.m.f of geometric distribution. Similarly, if r denotes the probability that a failed unit is repaired during $(i,$

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$i+1$); $i = 0, 1, 2, \dots$ then the probability that the unit will be repaired during $(t, t+1)$ is given by $r(1-r)^t$; $t = 0, 1, 2, \dots$. On the same way, the random variables representing the failure and repair times of RM may follow geometric distributions.

3 Notations and States of the System

a) Notations :

pq^x : P.m.f. of failure time of a unit ($p+q=1$).

rs^x : P.m.f. of repair time of a unit ($r+s=1$).

ab^x : P.m.f. of failure time of a RM ($a+b=1$).

cd^x : P.m.f. of repair time of a RM ($c+d=1$).

$q_{ij}(\cdot), Q_{ij}(\cdot)$: P.m.f. and C.d.f. of one step or direct transition time (T_{ij}) from state S_i to S_j .

p_{ij} : Steady state transition probability from state S_i to S_j .

$$p_{ij} = Q_{ij}(\infty)$$

$Z_i(t)$: Probability that the system sojourns in state S_i at epochs $0, 1, 2, \dots$, up to $(t-1)$.

ψ_i : Mean sojourn time in state S_i .

$*, h$: Symbol and dummy variable used in geometric transform e. g.

$$GT[q_{ij}(t)] = q_{ij}^*(h) = \sum_{t=0}^{\infty} h^t q_{ij}(t)$$

b) Symbols for the states of the systems:

N_o / N_s : Unit in normal (N) mode and operative/standby.

F_r / F_w : Unit in total failure (F) mode and under repair/waiting for repair.

RM_o / RM_g : RM in normal (N) mode and operative/good condition (non-functioning).

RM_r : RM in failure (F) mode and under repair.

With the help of above symbols the possible states (S_0 to S_5) of the system along with failure and repair rates against the possible transitions are shown in the transition diagram (Fig.1). In view of the discrete distributions involved, the stochastic model under study leads to the discrete parametric Markov-Chain with state space $(S_0, S_1, S_2, S_3, S_4, S_5)$.

4 Explanation of Transitions Between the States

System initially starts from state S_0 where one unit is operative and the other identical unit is kept into cold standby.

Also, in state S_0 the RM is good. From this state, the system approaches to state S_1 if the operating unit fails with rate p . In view of happening this event, the failed unit goes into repair with the help of RM and standby unit becomes operative instantaneously with the help of perfect switching device. Similarly, from state S_1 the following seven mutually exclusive transitions are possible-

i) Before the failures of operating unit and RM, the repair of the failed unit is completed with rate r , resulting the transition to state S_0 .

ii) Before the repair of failed unit and failure of RM, the operating unit is failed with rate p , resulting the transition to state S_3 .

iii) Before the repair of failed unit and failure of operating unit, the RM is failed with rate a , resulting the transition to state S_2 .

iv) Before the failures of RM, the repair of failed unit is completed and operating unit is failed at the same epoch with rates r, p resulting the transition to state S_1 again.

v) Before the failures of operating unit, the repair of the failed unit is completed and RM is failed at the same epoch with rates r, a resulting the transition to state S_4 .

vi) Before the repair of failed unit, operating unit and RM both are failed at same epoch with rates p, a resulting the transition to state S_5 .

vii) At the same epoch the repair of a failed unit is completed as well as the operating unit and RM are failed with rates r, p, a resulting the transition to state S_2 .

On the same way the moves of the system from other states can be observed.

5 Transition Probabilities

Let $Q_{ij}(t)$ be the probability that the system transits from state S_i to S_j during time interval $(0, t)$ i.e., if T_{ij} is the transition time from state S_i to S_j then

$$Q_{ij}(t) = P[T_{ij} \leq t]$$

By using simple probabilistic arguments, we have

$$Q_{01}(t) = (1 - q^{t+1})$$

$$Q_{10}(t) = \frac{bqr}{(1 - bqs)} \{1 - (bqs)^{t+1}\}$$

$$Q_{11}(t) = \frac{bpr}{(1 - bqs)} \{1 - (bqs)^{t+1}\}$$

$$Q_{12}(t) = \frac{a(qs + pr)}{(1 - bqs)} \{1 - (bqs)^{t+1}\}$$

TRANSITION DIAGRAM

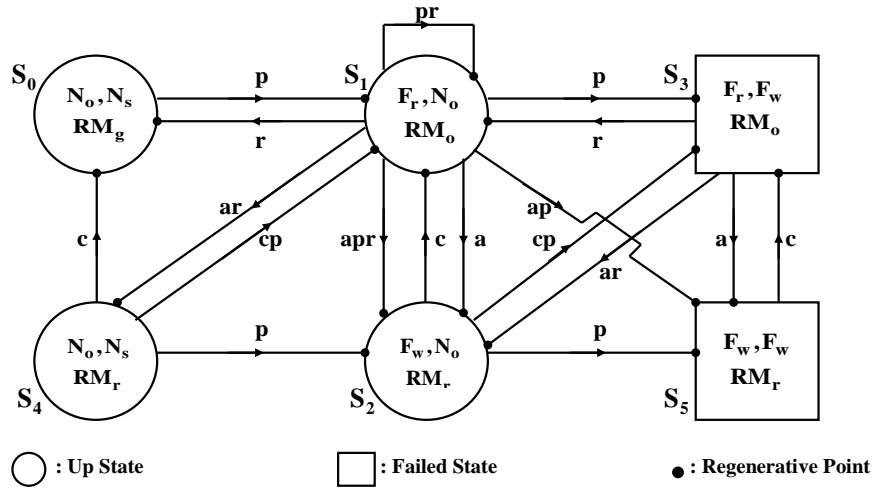


Fig.1

$$Q_{13}(t) = \frac{bps}{(1-bqs)} \{1-(bqs)^{t+1}\}$$

$$Q_{14}(t) = \frac{aqr}{(1-bqs)} \{1-(bqs)^{t+1}\}$$

$$Q_{15}(t) = \frac{aps}{(1-bqs)} \{1-(bqs)^{t+1}\}$$

$$Q_{21}(t) = \frac{cq}{(1-dq)} \{1-(dq)^{t+1}\}$$

$$Q_{23}(t) = \frac{cp}{(1-dq)} \{1-(dq)^{t+1}\}$$

$$Q_{25}(t) = \frac{dp}{(1-dq)} \{1-(dq)^{t+1}\}$$

$$Q_{31}(t) = \frac{br}{(1-bs)} \{1-(bs)^{t+1}\}$$

$$Q_{32}(t) = \frac{ar}{(1-bs)} \{1-(bs)^{t+1}\}$$

$$Q_{35}(t) = \frac{as}{(1-bs)} \{1-(bs)^{t+1}\}$$

$$Q_{40}(t) = \frac{cq}{(1-dq)} \{1-(dq)^{t+1}\}$$

$$Q_{41}(t) = \frac{cp}{(1-dq)} \{1-(dq)^{t+1}\}$$

$$Q_{42}(t) = \frac{dp}{(1-dq)} \{1-(dq)^{t+1}\}$$

$$Q_{53}(t) = (1-d^{t+1}) \tag{1-17}$$

The steady state transition probabilities from state S_i to S_j can be obtained from (1-17) by taking $t \rightarrow \infty$ as follows:

$$p_{01} = p_{53} = 1, \quad p_{10} = \frac{bqr}{(1-bqs)}$$

$$p_{11} = \frac{bpr}{(1-bqs)}, \quad p_{12} = \frac{a(qs+pr)}{(1-bqs)}$$

$$p_{13} = \frac{bps}{(1-bqs)}, \quad p_{14} = \frac{aqr}{(1-bqs)}$$

$$p_{15} = \frac{aps}{(1-bqs)}, \quad p_{21} = \frac{cq}{(1-dq)}$$

$$p_{23} = \frac{cp}{(1-dq)}, \quad p_{25} = \frac{dp}{(1-dq)}$$

$$p_{31} = \frac{br}{(1-bs)}, \quad p_{32} = \frac{ar}{(1-bs)}$$

$$p_{35} = \frac{as}{(1-bs)}, \quad p_{40} = \frac{cq}{(1-dq)}$$

$$p_{41} = \frac{cp}{(1-dq)}, \quad p_{42} = \frac{dp}{(1-dq)}$$

We observe that the following relations hold-

$$\begin{aligned} p_{01} &= p_{53} = 1 \\ p_{10} + p_{11} + p_{12} + p_{13} + p_{14} + p_{15} &= 1 \\ p_{21} + p_{23} + p_{25} &= 1 \\ p_{31} + p_{32} + p_{35} &= 1 \\ p_{40} + p_{41} + p_{42} &= 1 \end{aligned} \tag{18-22}$$

6 Mean Sojourn Times

Let ψ_i be the sojourn time in state S_i ($i = 0, 1, 2, 3, 4, 5$) then mean sojourn time in state S_i is given by

$$\psi_i = \sum_{t=1}^{\infty} P[T \geq t]$$

In particular,

$$\begin{aligned} \psi_0 &= \frac{q}{p}, & \psi_1 &= \frac{bqs}{(1-bqs)} \\ \psi_2 = \psi_4 &= \frac{dq}{(1-dq)} = \psi \text{ (say)} \\ \psi_3 &= \frac{bs}{(1-bs)}, & \psi_5 &= \frac{d}{c} \end{aligned} \quad (23-27)$$

7 Methodology for Developing Equations

In order to obtain various interesting measures of system effectiveness we develop the recurrence relations for reliability, availability and busy period of repairman as follows-

a) Reliability of the system-

Here we define $R_i(t)$ as the probability that the system does not fail up to t epochs $0, 1, 2, \dots, (t-1)$ when it is initially started from up state S_i . To determine it, we regard the failed states S_3 and S_5 as absorbing states. Now, the expressions for $R_i(t); i = 0, 1, 2, 4$; we have the following set of convolution equations.

$$\begin{aligned} R_0(t) &= q^t + \sum_{u=0}^{t-1} q_{01}(u)R_1(t-1-u) \\ &= Z_0(t) + q_{01}(t-1) \odot R_1(t-1) \end{aligned}$$

Similarly,

$$\begin{aligned} R_1(t) &= Z_1(t) + q_{10}(t-1) \odot R_0(t-1) + q_{11}(t-1) \odot R_1(t-1) \\ &\quad + q_{12}(t-1) \odot R_2(t-1) + q_{14}(t-1) \odot R_4(t-1) \\ R_2(t) &= Z_2(t) + q_{21}(t-1) \odot R_1(t-1) \\ R_4(t) &= Z_4(t) + q_{40}(t-1) \odot R_0(t-1) + q_{41}(t-1) \odot R_1(t-1) \\ &\quad + q_{42}(t-1) \odot R_2(t-1) \end{aligned} \quad (28-31)$$

Where,

$$Z_1(t) = b^t q^t s^t, \quad Z_2(t) = Z_4(t) = d^t q^t = Z(t)$$

b) Availability of the system-

Let $A_i(t)$ be the probability that the system is up at epoch $(t-1)$, when it initially starts from state S_i . By using simple probabilistic arguments, as in case of reliability the following recurrence relations can be easily developed for $A_i(t); i = 0$ to 5.

$$\begin{aligned} A_0(t) &= Z_0(t) + q_{01}(t-1) \odot A_1(t-1) \\ A_1(t) &= Z_1(t) + q_{10}(t-1) \odot A_0(t-1) + q_{11}(t-1) \odot A_1(t-1) \\ &\quad + q_{12}(t-1) \odot A_2(t-1) + q_{13}(t-1) \odot A_3(t-1) \\ &\quad + q_{14}(t-1) \odot A_4(t-1) + q_{15}(t-1) \odot A_5(t-1) \\ A_2(t) &= Z(t) + q_{21}(t-1) \odot A_1(t-1) + q_{23}(t-1) \odot A_3(t-1) \\ &\quad + q_{25}(t-1) \odot A_5(t-1) \\ A_3(t) &= q_{31}(t-1) \odot A_1(t-1) + q_{32}(t-1) \odot A_2(t-1) \end{aligned}$$

$$\begin{aligned} &\quad + q_{35}(t-1) \odot A_5(t-1) \\ A_4(t) &= Z(t) + q_{40}(t-1) \odot A_0(t-1) + q_{41}(t-1) \odot A_1(t-1) \\ &\quad + q_{42}(t-1) \odot A_2(t-1) \\ A_5(t) &= q_{53}(t-1) \odot A_3(t-1) \end{aligned} \quad (32-37)$$

Where,

The values of $Z_i(t); i = 0, 1$ and $Z(t)$ are same as given in section 6(a).

c) Busy period of repairman-

Let $B_i^m(t)$ and $B_i^r(t)$ be the respective probabilities that the repairman is busy at epoch $(t-1)$ in the repair of RM and units, when system initially starts from S_i . Using simple probabilistic arguments as in case of reliability, the recurrence relations for $B_i^j(t); i = 0$ to 5 can be easily developed as below. The dichotomous variable δ takes values 1 and 0 respectively for $j = m$ and r .

$$\begin{aligned} B_0^j(t) &= q_{01}(t-1) \odot B_1^j(t-1) \\ B_1^j(t) &= (1-\delta)Z_1(t) + q_{10}(t-1) \odot B_0^j(t-1) \\ &\quad + q_{11}(t-1) \odot B_1^j(t-1) + q_{12}(t-1) \odot B_2^j(t-1) \\ &\quad + q_{13}(t-1) \odot B_3^j(t-1) + q_{14}(t-1) \odot B_4^j(t-1) \\ &\quad + q_{15}(t-1) \odot B_5^j(t-1) \\ B_2^j(t) &= \delta Z(t) + q_{21}(t-1) \odot B_1^j(t-1) + q_{23}(t-1) \odot B_3^j(t-1) \\ &\quad + q_{25}(t-1) \odot B_5^j(t-1) \\ B_3^j(t) &= (1-\delta)Z_3(t) + q_{31}(t-1) \odot B_1^j(t-1) \\ &\quad + q_{32}(t-1) \odot B_2^j(t-1) + q_{35}(t-1) \odot B_5^j(t-1) \\ B_4^j(t) &= \delta Z(t) + q_{40}(t-1) \odot B_0^j(t-1) + q_{41}(t-1) \odot B_1^j(t-1) \\ &\quad + q_{42}(t-1) \odot B_2^j(t-1) \\ B_5^j(t) &= \delta Z_5(t) + q_{53}(t-1) \odot B_3^j(t-1) \end{aligned} \quad (38-43)$$

Where,

$Z_1(t)$ and $Z(t)$ have the same values as in section 6(a)

and

$$Z_3(t) = b^t s^t, \quad Z_5(t) = d^t.$$

8 Analysis of Characteristics

a) Reliability and MTSF-

Taking geometric transforms of (28-31) and simplifying the resulting set of algebraic equations for $R_0^*(h)$, we get

$$R_0^*(h) = \frac{N_1(h)}{D_1(h)} \quad (44)$$

Where,

$$\begin{aligned} N_1(h) &= \left[1 - hq_{11}^* - hq_{21}^* (hq_{12}^* + h^2 q_{14}^* q_{42}^*) - h^2 q_{14}^* q_{41}^* \right] Z_0^* \\ &\quad + hq_{01}^* Z_1^* + hq_{01}^* (hq_{12}^* + hq_{14}^* + h^2 q_{14}^* q_{42}^*) Z^* \end{aligned}$$

$$D_1(h) = \left[1 - hq_{11}^* - hq_{21}^* (hq_{12}^* + h^2 q_{14}^* q_{42}^*) - h^2 q_{14}^* q_{41}^* \right. \\ \left. - h^2 q_{01}^* q_{10}^* - h^3 q_{01}^* q_{14}^* q_{40}^* \right]$$

Collecting the coefficient of h^1 from expression (44), we can get the reliability of the system $R_0(t)$. The MTSF is given by-

$$E(T) = \lim_{h \rightarrow 1} \sum_{t=1}^{\infty} h^t R(t) = \frac{N_1(1)}{D_1(1)} - 1 \quad (45)$$

$$N_1(1) = \left[1 - p_{11} - p_{21}(p_{12} + p_{14}p_{42}) - p_{14}p_{41} \right] \Psi_0 \\ + \Psi_1 + \left[p_{12} + p_{14}(1 + p_{42}) \right] \Psi$$

$$D_1(1) = \left[1 - p_{11} - p_{21}(p_{12} + p_{14}p_{42}) - p_{14}p_{41} \right] - p_{10} - p_{14}p_{40}$$

b) Availability Analysis-

On taking geometric transform of (32-37) and simplifying the resulting equations, we get

$$A_0^*(h) = \frac{N_2(h)}{D_2(h)} \quad (46)$$

Where,

$$N_2(h) = \left\{ 1 - h^2 q_{35}^* q_{53}^* - hq_{32}^* (hq_{23}^* + h^2 q_{25}^* q_{53}^*) \right\} \left\{ h^2 q_{01}^* q_{14}^* Z^* \right. \\ \left. + (1 - hq_{11}^* - h^2 q_{14}^* q_{41}^*) Z_0^* + hq_{01}^* Z_1^* \right\} \\ + \left\{ (1 - h^2 q_{35}^* q_{53}^*) (hq_{12}^* + h^2 q_{14}^* q_{42}^*) + hq_{32}^* (hq_{13}^* \right. \\ \left. + h^2 q_{15}^* q_{53}^*) \right\} \left\{ hq_{01}^* Z^* - hq_{21}^* Z_0^* \right\} - hq_{31}^* \left\{ hq_{13}^* \right. \\ \left. + h^2 q_{15}^* q_{53}^* + (hq_{23}^* + h^2 q_{25}^* q_{53}^*) (hq_{12}^* + h^2 q_{14}^* q_{42}^*) \right\} Z_0^*$$

$$D_2(h) = \left\{ 1 - hq_{11}^* - h^2 q_{14}^* q_{41}^* - h^2 q_{01}^* q_{10}^* + h^3 q_{01}^* q_{14}^* q_{40}^* \right\} \\ \left\{ 1 - h^2 q_{35}^* q_{53}^* - hq_{32}^* (hq_{23}^* + h^2 q_{25}^* q_{53}^*) \right\} \\ - \left\{ (1 - h^2 q_{35}^* q_{53}^*) (hq_{12}^* + h^2 q_{14}^* q_{42}^*) + hq_{32}^* (hq_{13}^* \right. \\ \left. + h^2 q_{15}^* q_{53}^*) \right\} \left\{ h^2 q_{01}^* q_{20}^* + hq_{21}^* \right\} - hq_{31}^* \left\{ hq_{13}^* \right. \\ \left. + h^2 q_{15}^* q_{53}^* + (hq_{23}^* + h^2 q_{25}^* q_{53}^*) (hq_{12}^* + h^2 q_{14}^* q_{42}^*) \right\}$$

The steady state availability of the system is given by-

$$A_0 = \lim_{t \rightarrow \infty} A_0(t) = \lim_{h \rightarrow 1} (1-h) \frac{N_2(h)}{D_2(h)}$$

As $D_2(h)$ at $h=1$ is zero, therefore by applying L. Hospital rule, we get

$$A_0 = - \frac{N_2'(1)}{D_2'(1)} \quad (47)$$

Where,

$$N_2(1) = (p_{31} + p_{32}p_{21}) \left\{ (p_{10} + p_{14}p_{40}) \Psi_0 + \Psi_1 \right\} + \left\{ (1 - p_{35}) \right. \\ \left. (p_{12} + p_{14}p_{42}) + p_{32}(p_{13} + p_{15}) + p_{14}(p_{31} + p_{32}p_{21}) \right\} \Psi$$

and

$$D_2'(1) = (p_{31} + p_{32}p_{21}) \left\{ (p_{10} + p_{14}p_{40}) \Psi_0 + \Psi_1 \right\} + \left\{ (1 - p_{35}) \right.$$

$$\left. (p_{12} + p_{14}p_{42}) + p_{32}(p_{13} + p_{15}) + p_{14}(p_{31} + p_{32}p_{21}) \right\} \Psi \\ + \left\{ (1 - p_{21})(p_{12} + p_{14}p_{42}) + (p_{13} + p_{15}) \right\} \Psi_3 \\ + \left\{ (p_{35} + p_{32}p_{25})(p_{12} + p_{13} + p_{15} + p_{14}p_{42}) - (p_{12} \right. \\ \left. + p_{14}p_{42}) - (p_{21}p_{35} + p_{31}p_{25}) + p_{15}(p_{31} + p_{32}p_{21}) \right\} \Psi_5$$

Now the expected up time of the system up to epoch (t-1) is given by

$$\mu_{up}(t) = \sum_{x=0}^{t-1} A_0(x)$$

so that

$$\mu_{up}^*(h) = A_0^*(h)/(1-h) \quad (48)$$

c) Busy Period Analysis-

On taking geometric transforms of (38-43) and simplifying the resulting equations for $j = m$ and r , we get

$$B_0^{m*}(h) = \frac{N_3(h)}{D_2(h)} \quad \text{and} \quad B_0^{r*}(h) = \frac{N_4(h)}{D_2(h)} \quad (49-50)$$

Where,

$$N_3(h) = hq_{01}^* \left[(1 - h^2 q_{35}^* q_{53}^*) (hq_{12}^* + hq_{14}^* + h^2 q_{14}^* q_{42}^*) \right. \\ \left. + hq_{32}^* \left\{ (hq_{13}^* + h^2 q_{15}^* q_{53}^*) - hq_{14}^* (hq_{23}^* + h^2 q_{25}^* q_{53}^*) \right\} \right] Z^* \\ + hq_{01}^* \left\{ (hq_{15}^* + h^2 q_{13}^* q_{35}^*) + hq_{14}^* (h^2 q_{42}^* q_{25}^* + h^3 q_{23}^* q_{35}^* q_{42}^*) \right. \\ \left. + hq_{12}^* (h^2 q_{23}^* q_{35}^* + hq_{25}^*) + hq_{32}^* (h^2 q_{13}^* q_{25}^* - h^2 q_{15}^* q_{23}^*) \right\} Z_5^* \\ N_4(h) = hq_{01}^* \left[\left\{ 1 - h^2 q_{35}^* q_{53}^* - hq_{32}^* (hq_{23}^* + h^2 q_{25}^* q_{53}^*) \right\} Z_1^* + \left\{ (hq_{13}^* \right. \right. \\ \left. \left. + h^2 q_{15}^* q_{53}^*) + (hq_{23}^* + h^2 q_{25}^* q_{53}^*) (hq_{12}^* + h^2 q_{14}^* q_{42}^*) \right\} Z_3^* \right]$$

and $D_2(h)$ is same as in availability analysis.

In the long run, the respective probabilities that the repairman is busy in the repair of RM and units are given by-

$$B_0^m = \lim_{t \rightarrow \infty} B_0^m(t) = \lim_{h \rightarrow 1} (1-h) \frac{N_3(h)}{D_2(h)}$$

$$B_0^r = \lim_{t \rightarrow \infty} B_0^r(t) = \lim_{h \rightarrow 1} (1-h) \frac{N_4(h)}{D_2(h)}$$

But $D_2(h)$ at $h=1$ is zero, therefore by applying L. Hospital rule, we get

$$B_0^m = - \frac{N_3'(1)}{D_2'(1)} \quad \text{and} \quad B_0^r = - \frac{N_4'(1)}{D_2'(1)} \quad (51-52)$$

Where,

$$N_3(1) = \left\{ (1 - p_{35})(p_{12} + p_{14}p_{42}) + p_{32}(p_{13} + p_{15}) + p_{14}(p_{31} \right. \\ \left. + p_{32}p_{21}) \right\} \Psi + \left\{ (p_{35} + p_{32}p_{25})(p_{12} + p_{13} + p_{15} + p_{14}p_{42}) \right. \\ \left. + p_{15}(p_{31} + p_{32}p_{21}) - (p_{12} + p_{14}p_{42})(p_{21}p_{35} + p_{31}p_{25}) \right\} \Psi_5$$

$$N_4(1) = (p_{31} + p_{32}p_{21}) \Psi_1 + \left\{ (1 - p_{21})(p_{12} + p_{14}p_{42}) \right. \\ \left. + (p_{13} + p_{15}) \right\} \Psi_3$$

and $D_2'(1)$ is same as in availability analysis.

The expected busy periods of the repairman in the repair of RM and units up to epoch (t-1) are respectively given by-

$$\mu_b^m(t) = \sum_{x=0}^{t-1} B_0^m(x)$$

$$\mu_b^r(t) = \sum_{x=0}^{t-1} B_0^r(x)$$

So that,

$$\mu_b^{m*}(h) = \frac{B_0^{m*}(h)}{(1-h)} \quad (53)$$

$$\mu_b^{r*}(h) = \frac{B_0^{r*}(h)}{(1-h)} \quad (54)$$

9 Profit Function Analysis

We are now in the position to obtain the net expected profit incurred up to epoch (t-1) by considering the characteristics obtained in earlier sections.

Let us consider,

K_0 = revenue per-unit time by the system when it is operative.

K_1 = cost per-unit time when repairman is busy in the repair of the failed RM.

K_2 = cost per-unit time when repairman is busy in the repair of the failed units.

Then, the net expected profit incurred up to epoch (t-1) is given by

$$P(t) = K_0\mu_{up}(t) - K_1\mu_b^m(t) - K_2\mu_b^r(t) \quad (55)$$

The expected profit per unit time in steady state is as follows-

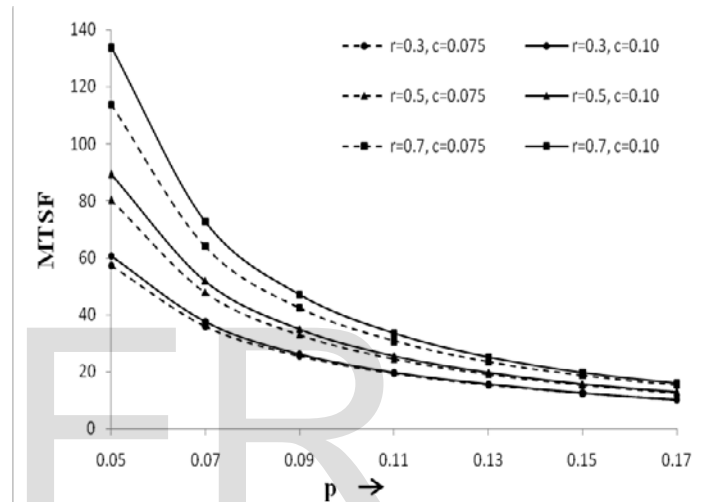
$$\begin{aligned}
 P &= \lim_{t \rightarrow \infty} \frac{P(t)}{t} \\
 &= K_0 \lim_{h \rightarrow 1} (1-h)^2 \frac{A_0^*(h)}{(1-h)} - K_1 \lim_{h \rightarrow 1} (1-h)^2 \frac{B_0^{m*}(h)}{(1-h)} \\
 &\quad - K_2 \lim_{h \rightarrow 1} (1-h)^2 \frac{B_0^{r*}(h)}{(1-h)} \\
 &= K_0 A_0 - K_1 B_0^m - K_2 B_0^r \quad (56)
 \end{aligned}$$

10 Graphical Representation

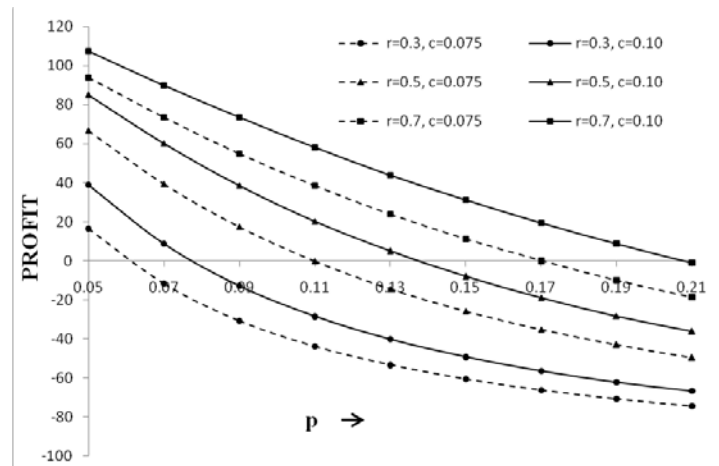
The curves for MTSF and profit function have been drawn for different values of parameters p, r, c. Fig. 2 depicts the variations in MTSF with respect to failure rate (p) of an operative unit for different values of repair rate (r = 0.3, 0.5, 0.7) of a failed unit and the failure rate (c = 0.075, 0.10) of RM. From the curves we observe that MTSF decreases uniformly as the values of p increase. It also reveals that the MTSF increases with the increase in r and increases with the increase in c.

Similarly, fig. 3 reveals the variations in profit (P) with respect to p for varying values of r and c, when the values of other parameters are kept fixed as a = 0.6, $K_0=150$, $K_1=100$ and $K_2=80$. From the curves we observe that profit decreases uniformly as the values of p increase. It also reveals that the profit increases with the increase in r and increases with the increase in c. From this figure it is clear from the dotted curves that the system is profitable only if failure rate (p) is greater than 0.06, 0.11 and 0.17 respectively for r = 0.3, 0.5, and 0.7 for fixed value of c = 0.075. From smooth curves, we conclude that the system is profitable only if p is greater than 0.78, 0.14 and 0.21 respectively for r = 0.3, 0.5, and 0.7 for fixed value of c = 0.10.

Behaviour of MTSF with respect to p, r and c



Behaviour of Profit (P) with respect to p, r and c



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